

Worked Solutions

Edexcel C4 Paper H

1. (a)

x	-1	0	1
$\frac{1}{1+e^{-1}}$	$\frac{1}{1+e}$	$\frac{1}{1+1}$	$\frac{1}{1+\frac{1}{e}}$

$$\frac{1}{1+\frac{1}{e}} = \frac{e}{e+1}$$

$$\begin{aligned} \text{integral} &\approx \frac{1}{2} \left[\frac{1}{1+e} + \frac{e}{e+1} + 2 \times \frac{1}{2} \right] \\ &= \frac{1}{2} \left[\frac{1+e+1+e}{1+e} \right] = 1 \end{aligned}$$

(b) let $I = \int_{-1}^1 \frac{1}{1+e^{-x}} dx$

put $u = e^x$
 $\frac{du}{dx} = e^x$
 $du = e^x dx$

$$= \int_{-1}^1 \frac{e^x}{e^x+1} dx$$

when $x = 1, u = e$
 $x = -1, u = e^{-1}$

$$\therefore I = \int_{e^{-1}}^e \frac{du}{u+1} = \left[\ln(u+1) \right]_{e^{-1}}^e$$

$$= \ln(e+1) - \ln\left(1 + \frac{1}{e}\right)$$

$$= \ln\left(\frac{e+1}{1+\frac{1}{e}}\right) = \ln\left[\frac{(1+e)e}{(e+1)}\right] = \ln e = 1 \quad (4)$$

2. (a) (i) differentiating implicitly, $1 = e^y \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

(ii) when $y = 0, x = e^0 = 1$ $\frac{dy}{dx} = 1$

equation of tangent is $y - 0 = x - 1$
 $y = x - 1$

(b) $x = \sin y$ $1 = \cos y \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$

3. (a) $\frac{dy}{dx} = \frac{-2 \sin \theta}{2 \cos \theta} = -\frac{\sin \theta}{\cos \theta}$

equation of tangent is $y - (2 \cos \theta + 2) = -\frac{\sin \theta}{\cos \theta}$

$$y \cos \theta - 2 \cos^2 \theta - 2 \cos \theta = -x \sin \theta + 2 \sin^2 \theta + 2 \sin \theta$$

$$x \sin \theta + y \cos \theta = 2 \cos^2 \theta + 2 \sin^2 \theta + 2 \sin \theta - 2 \cos \theta$$

(b) when $\theta = \frac{\pi}{2}$ tangent is $x + 0 = 2 + 0 + 1$

$$x = 3$$

(c) $\sin \theta = \frac{x-1}{2}, \cos \theta = \frac{y-2}{2}$

$$\left(\frac{x-1}{2}\right)^2 + \left(\frac{y-2}{2}\right)^2 = 1 \quad [\sin^2 \theta + \cos^2 \theta = 1]$$

$$(x-1)^2 + (y-2)^2 = 4$$

4. (a) $\int (y+1)dy = -\int (x-2)dx$

$$\frac{1}{2}(y+1)^2 = -\frac{1}{2}(x-2)^2 + k$$

(2, 2) lies on C, $\therefore \frac{1}{2}9 = -\frac{1}{2} \times 0 + k$

C is $\frac{1}{2}(y+1)^2 + \frac{1}{2}(x-2)^2 = \frac{9}{2}$

or $(x-2)^2 + (y+1)^2 = 9$ (6)

(b) Circle centre (2, -1), radius 3 (2)

5. (a) $t = 0, \theta = 70 + 2 = 72$ (1)

(b) $\theta = 70e^{-1} + 2 = 27.8$ (2)

(c) as $t \rightarrow \infty, e^{-0.1t} \rightarrow 0$
 $\therefore \theta \rightarrow 2$ (2)

(d) $10 = 70e^{-0.1t} + 2$
 $e^{-0.1t} = \frac{8}{70}$
 $-0.1t = \ln \frac{8}{70}, t = 21.7$ minutes (3)

6. (a) divide each term by $\cos^2 \theta$,

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$\tan^2 \theta + 1 = \sec^2 \theta$ (2)

(b) $I = \int_{\frac{1}{\sqrt{3}}}^1 \frac{1}{1+x^2} dx$

let $x = \sec \theta$
 $x = 1, \theta = \frac{\pi}{4}$
 $x = \frac{1}{\sqrt{3}}, \theta = \frac{\pi}{6}$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec^2 \theta}{\sec^2 \theta} d\theta = \left[\theta \right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{\pi}{12}$$

7. (a) $\frac{9x}{(1-2x)(1+x)^2} \equiv \frac{A}{1-2x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$

$$9x \equiv A(1+x)^2 + B(1-2x)(1+x)$$

$x = \frac{1}{2}: \frac{9}{2} = A \cdot \left(\frac{3}{2}\right)^2 \Rightarrow A = 2$

$x = -1: -9 = C(1+2) \Rightarrow C = -3$

constants: $0 = A + B + C \Rightarrow B = 1$

\therefore expression is $\frac{2}{1-2x} + \frac{1}{1+x} - \frac{3}{(1+x)^2}$

(b) $2(1-2x)^{-1} + (1+x)^{-1} - 3(1+x)^{-2}$
 $= 2 \left[1 + (-1)(-2x) + \frac{(-1)(-2)}{2} (-2x)^2 + \frac{(-1)(-2)(-3)}{6} (-2x)^3 \right]$
 $+ \left[1 - x + \frac{(-1)(-2)}{2} (x^2) + \frac{(-1)(-2)(-3)}{3 \cdot 2} (-x)^3 \right]$
 $- 3 \left[1 + (-2)x + \frac{(-2)(-3)}{2} x^2 + \frac{(-2)(-3)(-3)}{3 \cdot 2} x^3 \right]$
 $= (2 + 4x + 8x^2 + 16x^3) + (1 - x + x^2 - x^3) -$
 $= 9x + 27x^3$

$$8. (a) \text{ area} = \int_1^3 \left(2 + \frac{1}{x}\right) dx = [2x + \ln x]_1^3 = 4 + \ln 3 \quad (3)$$

$$\begin{aligned} (b) \text{ volume} &= \pi \int_1^3 \frac{4x^2 + 4x + 1}{x^2} dx \\ &= \pi \int_1^3 \left(4 + \frac{4}{x} + x^{-2}\right) dx \\ &= \pi \left[4x + 4 \ln x - \frac{1}{x}\right]_1^3 \\ &= \pi \left[12 + 4 \ln 3 - \frac{1}{3} - (4 + 0 - 1)\right] = \pi \left[\frac{26}{3} + 4 \ln 3\right] \quad (6) \end{aligned}$$

$$9. (a) \vec{AB} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \text{ line through } AB \text{ is } r = \begin{pmatrix} 7 \\ 8 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \quad (3)$$

$$(b) \vec{AO} = \begin{pmatrix} -7 \\ -8 \\ 0 \end{pmatrix} \quad |\vec{AO}| = \sqrt{49 + 64} = \sqrt{113}$$

$$\vec{AB} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \quad |\vec{AB}| = \sqrt{4 + 1 + 9} = \sqrt{14}$$

$$\begin{pmatrix} -7 \\ -8 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \sqrt{113} \sqrt{14} \cos \theta,$$

$$-14 + 8 = \sqrt{113} \sqrt{14} \cos \theta,$$

$$\theta = 98.7^\circ$$

acute angle between OA and AB is 81° (nearest 1°)

(c) M lies on line AB

$$\therefore \vec{OM} \text{ is } \begin{pmatrix} 7 + 2\lambda \\ 8 - \lambda \\ 0 + 3\lambda \end{pmatrix}$$

$$\vec{OM} \cdot \vec{AB} = 0$$

$$\begin{pmatrix} 7 + 2\lambda \\ 8 - \lambda \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

$$= 14 + 4\lambda - 8 + \lambda + 9\lambda = 0$$

$$\lambda = \frac{-3}{7}$$

$$\text{position vector of } M \text{ is } \begin{pmatrix} 7 - \frac{6}{7} \\ 8 + \frac{3}{7} \\ -9 \\ \frac{3}{7} \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 43 \\ 59 \\ -9 \end{pmatrix}$$

